Enrollment No: ______________________ **Exam Seat No:**______________________

C.U.SHAH UNIVERSITY

Summer Examination-2022

Subject Name: Ring Theory

Subject Code: 4SC06RIT1 **Branch: B.Sc. (Mathematics)**

Semester: 6 **Date: 05/05/2022** **Time: 02:30 To 05:30** **Marks: 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) The characteristic of ring $(Z_7, +_7, \times_7)$ is______. (01)
(a) 5 (b) 6 (c) 7 (d) 8

b) A commutative ring with unity and without zero-divisors is called_______. (01)
(a) Field (b) Integral domain (c) Division ring (d) None

c) If $a^2 = a$, $\forall a \in R$ then the ring $(R, +, \cdot)$ is called______. (01)
(a) Division ring (b) Boolean ring (c) Ring with unity (d) Integral domain

d) The total number of zero-divisors in ring $(Z_5, +_5, \times_5)$ ______. (01)
(a) 1 (b) 2 (c) 5 (d) 0

e) The characteristic of ring $(Z_4, +_4, \times_4)$ is _______. (01)
(a) 1 (b) 2 (c) 3 (d) 4

f) A non-zero element a and b of ring $(R, +, \cdot)$ are called zero divisors, if _____. (01)
(a) $ab = 0$ (b) $ab \neq 0$ (c) $ab = 1$ (d) $ab \neq 1$

g) Every integral domain is not a______. (01)
(a) Ring (b) Commutative ring
(c) Field (d) Abelian group with respect to addition

h) Which of the following is correct? (01)
(a) An integral domain is field (b) A field is an integral domain
(c) A field has zero-divisors (d) all

i) A non-zero element a and b of ring $(R, +, \cdot)$ are called units , if _____. (01)
(a) $ab = 0$ (b) $ab \neq 0$ (c) $ab = 1$ (d) $ab \neq 1$

j) $x^2 - 2$ is reducible over _____. (01)
(a) N (b) Z (c) Q (d) R

k) Define: Subring of ring. (01)

l) Define: Ideal in ring $(R, +, \cdot)$. (01)

m) Define: Monic polynomial in $R[x]$. (01)

n) State: Division algorithm in ring polynomial $R[x]$. (01)

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) Let $M = \{(x, y) | x, y \in R\}$ for $(a, b), (c, d) \in M$ where the addition and multiplication defined as $(a, b) + (c, d) = (a + c, b + d)$ and $(a, b), (c, d)$ and (06)

		$(a,b)\cdot(c,d) = (ac,bd)$ then shw that $(M,+,\cdot)$ is commutative ring.	
	b)	Prove that R is ring without zero-divisors if and only if cancellation law holds in R.	(04)
	c)	If $(R,+,\cdot)$ is ring then show that (i) $0a = 0 = a0$ (ii) $a(-b) = -(ab) = (-a)b$	(04)
Q-3		**Attempt all questions**	**(14)**
	a)	$(R,+,\cdot)$ is a ring , if $(a+b)^2 = a^2 + 2ab + b^2$, $\forall a,b \in R$ then show that $(R,+,\cdot)$ is commutative ring.	(05)
	b)	State and prove necessary and sufficient condition for non-empty subset to be a subring.	(05)
	c)	$M_2(R)$ is ring of all 2×2 matrices and $S = \left\{\begin{bmatrix} a & 0 \\ 0 & b\end{bmatrix} \middle\vert a,b \in R\right\}$ then show that S is sub ring of $M_2(R)$.	(04)
Q-4		**Attempt all questions**	**(14)**
	a)	$(Z,+,\cdot)$ is a ring and $S = \{3m \mid m \in Z\}$ then show that S is subring of $(Z,+,\cdot)$.	(05)
	b)	If S_1 and S_2 are two subring of ring R then prove that $S_1 \cap S_2$ is also subring of ring R.	(05)
	c)	R is ring with unit element , a positive integer n is characteristic of ring R if and only if $n\cdot 1 = 0$.	(04)
Q-5		**Attempt all questions**	**(14)**
	a)	Prove that every field is an integral domain but converse is not true.	(05)
	b)	Let M be a ring of 2×2 matrices over integers and set $I = \left\{\begin{bmatrix} a & 0 \\ b & 0\end{bmatrix} \middle\vert a,b \in R\right\}$ then show that I is left Ideal of M but not right Ideal of M.	(05)
	c)	Write down all principal ideal of $(Z_8, +_8, \times_8)$.	(04)
Q-6		**Attempt all questions**	**(14)**
	a)	Prove that every finite integral domain is field.	(05)
	b)	$\emptyset: R \to R'$ is homomorphism then show that $Ker\emptyset$ is subring of R.	(05)
	c)	If $\emptyset: R \to R'$ is homomorphism then show that (i) $\emptyset(0) = 0'$ (ii) $\emptyset(-a) = -\emptyset a$ $\forall a \in R$ where $0'$ is the additive identity of ring R'.	(04)
Q-7		**Attempt all questions**	**(14)**
	a)	For a non-zero polynomial $f(x), g(x) \in D[x]$, then prove that $[f+g] = [f] + [g]$.	(05)
	b)	If $f(x)$ and $g(x)$ are monic polynomial in $R[x]$ then show that $f(x)g(x)$ also monic polynomial.	(05)
	c)	Suppose that $f = (0,1,0,2,0,0,0 \ldots.)$ and $g = (1,0,-3,1,0,0,0,\ldots\ldots)$ then find $f+g$ and fg.	(04)
Q-8		**Attempt all questions**	**(14)**
	a)	State and prove division algorithm in ring polynomial $R[x]$.	(07)
	b)	Suppose $p(x), f(x), g(x) \in R[x]$, if $p(x)$ is divisor of both $f(x)$ and $g(x)$ then $p(x)$ is also divisor of $f(x)a(x) + g(x)b(x)$ for some $a(x), b(x) \in R[x]$.	(04)
	c)	Obtain a quotient and reminder when $f(x) = 4x^4 - 3x^2 + 2$ divided by $g(x) = x^3 - 2x + 1$.	(03)